MASSACHUSETTS INSTITUTE of TECHNOLOGY
LIBRARIES

Digitized by the Internet Archive
in 2011 with funding from
Boston Library Consortium Member Libraries

http://www.archive.org/details/cheaptalkcanmatt00farr

HB31
.M415
no.482

MASS. INST. TECH.
JUL 14 1988
LIBRARIES

working paper
department
of economics

CHEAP TALK CAN MATTER IN BARGAINING

by

Joseph Farrell
Robert Gibbons

No. 482 February 1988

massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139

# CHEAP TALK CAN MATTER IN BARGAINING

by

Joseph Farrell
Robert Gibbons

No. 482 February 1988

M.I.T. LIBRARIES
JUL 15 1988
RECEIVED

## Abstract

This paper describes an intuitive way in which cheap talk can matter in a two-stage bargaining game in which talk may be followed by serious negotiation. The intuition that all buyers would claim to have low reservation prices is incorrect in our model. Instead, the paper emphasizes that if good-faith participation is endogenously determined then the parties can use talk to trade off bargaining position against the probability of continued negotiation. Our cheap-talk equilibrium features bargaining behavior that could not be equilibrium behavior in the absence of talk.

# Cheap Talk Can Matter in Bargaining*

July 1987
Revised February 1988

Joseph Farrell
Department of Economics
U.C. Berkeley
Berkeley, CA 94720

Robert Gibbons
Department of Economics
M.I.T.
Cambridge, MA 02139

*We thank Peter Cramton, Steven Matthews, William Samuelson, two referees, and seminar audiences at Berkeley, Chicago, Duke, M.I.T., the 1987 NSF Decentralization Conference, and Stanford for helpful comments. Research support through NSF grants IRI-8712238 and IST-8609691 is gratefully acknowledged.

# Cheap Talk Can Matter in Bargaining*

by

Joseph Farrell**

and

Robert Gibbons***

July 1987, Revised: February 1988

* We thank Peter Cramton, Steven Matthews, William Samuelson, two referees, and seminar audiences at Berkeley, Chicago, Duke, M.I.T., the 1987 NSF Decentralization Conference, and Stanford for helpful comments. Research support through NSF grants IRI-8712238 and IST-8609691 is gratefully acknowledged.

** Department of Economics, University of California, Berkeley.

***Department of Economics, Massachusetts Institute of Technology.

## 1. Introduction

Since the seminal work of Spence (1974), economists have understood how an informed agent's choices may reveal private information if they affect him differently depending on what he knows. This idea of costly signaling has been among the most influential in economic theory in over a decade, underlying analyses of everything from education to entry-deterrence. But perhaps its greatest influence has been in the modern noncooperative theory of bargaining: a large and growing literature, beginning with Fudenberg and Tirole (1983) and Sobel and Takahashi (1983), analyzes how bargainers can improve their terms of trade by undertaking costly actions (notably delay) intended to convince the other party that their interest in trade is at best lukewarm.

While communication through costly signals is undoubtedly important, the continuing popularity of language is (to us) persuasive evidence that much communication also occurs through costless words, or cheap talk. Talk is ubiquitous and is often listened to, even where no real penalty attaches to lying, and where claims have no direct impact on payoffs. Communication through such cheap talk does not fit directly in the Spence-style costly-signaling framework, but Crawford and Sobel (1982) showed formally that cheap talk can be credible in equilibrium if the parties have at least some interests in common.

One might expect that cheap talk could not matter in bargaining because each side has an incentive to seem unenthusiastic, as in a bazaar where the buyer pours scorn on the seller's goods, which the latter declares are so precious as to be scarcely for sale. Yet casual observation reveals much cheap talk of the opposite kind: people often claim to be "seriously

interested" in trading. Unless all this is mere meaningless noise, something is missing in the theory.

One possibility is that these claims -- or confessions -- of urgent desire to trade are meant to encourage the other side to participate in more detailed negotiation. As we will show below, if a claim that one is "keen" makes one's partner more likely to negotiate, then it is the keenest types (high-value buyers, low-value sellers) who are most willing to send such a message, damaging as it is to their terms of trade if trade occurs.

The following story illustrates the element of common interest that drives our analysis: cheap talk can affect whether negotiation ensues. Imagine that one Saturday evening, two corporate moguls have a chance encounter at their country club. One mogul's company owns a division that the other mogul's firm may wish to buy. Serious negotiation, involving binding offers and hordes of lawyers, can take place on Monday morning; all that can happen Saturday night is talk. If, based on this talk, the moguls conclude that there is sufficient prospect of gains from trade, then they will send their lawyers into the fray on Monday morning. Otherwise, Saturday evening will be the end of it.

In this paper, we turn this basic intuition into a precise equilibrium statement in a particular bargaining model, the sealed-bid double auction studied by Chatterjee and Samuelson (1983). We analyze a two-stage game: talk comes first, then formal (binding) negotiation. Not only is information conveyed by cheap talk in equilibrium, but the equilibrium mapping from the buyer's and seller's reservation values to outcomes (whether trade occurs and if so at what price) differs from any that could occur in an equilibrium without talk. Further, some outcomes of the "talk" stage induce second-stage negotiation behavior that could not be equilibrium behavior absent the

changes in beliefs that the talk causes. Matthews and Postlewaite (1987) also analyze cheap talk preceding a sealed-bid double auction. We discuss their work in Section 2.

To complete this Introduction, we describe the differences between our cheap-talk equilibrium and the related literatures on bargaining and on mechanism design. As described above, in most game-theoretic analyses of bargaining, communication takes place only through actions that can directly affect payoffs, and that therefore can be costly signals. Typically, such an action either directly imposes costs of delay, or directly affects payoffs by constituting an offer that is binding if the other player accepts it, or both. Cheap talk does neither of these things. Of course, in equilibrium in a cheap-talk game, different types have incentives to choose different cheap-talk messages, but no part of these incentives consists of exogenous costs or benefits. This differentiates cheap talk from signaling, and differentiates our analysis from a standard bargaining game.

Cheap talk also differs from a mechanism (in the sense of Myerson and Satterthwaite (1983)). In such a mechanism, messages without direct costs are used, but a mediator controls the communication and is committed to enforcing a given outcome as a function of the messages. In cheap-talk equilibrium, by contrast, no agent can commit himself to a choice of outcome as a function of messages; rather, the outcome must be a perfect Bayesian equilibrium given the information conveyed by the messages. Moreover, these messages become common knowledge, whereas a Myerson-Satterthwaite mediator can, and typically does, limit the information he passes on to the players. Of course, every cheap-talk equilibrium can be implemented as a mechanism, but the reverse is not true.

## 2. A Cheap-Talk Equilibrium

This section analyzes cheap talk in a well-known model of bargaining under incomplete information. In terms of our story of the two corporate moguls, if the parties do meet on Monday, they play the following sealed-bid double auction. Buyer and seller name prices $p_b$ and $p_s$ respectively, and trade takes place at price $(p_b+p_s)/2$ if $p_b \geq p_s$; otherwise, there is no trade.[1]

Chatterjee and Samuelson (1983) analyze a class of equilibria of this double auction without cheap talk. They show that bounded, strictly monotone, and differentiable equilibrium strategies must satisfy a linked pair of differential equations. In the standard case in which $v_s$ and $v_b$ are independently and uniformly distributed on $[0,1]$, these differential equations have a solution in which both the buyer and the seller play linear strategies. We call this the Chatterjee-Samuelson equilibrium and we use it to define equilibrium behavior whenever possible. There are, however, many other equilibria in a double auction in the absence of cheap talk, only some of which satisfy the conditions that Chatterjee and Samuelson assumed. Leininger, Linhart, and Radner (1986) and Satterthwaite and Williams (1987) explore some of these alternative equilibria, and we use one in our model when it is not possible to use the Chatterjee-Samuelson equilibrium.

Before the double-auction stage of our two-stage game there is a cheap-talk stage. (On Saturday the moguls can engage in cheap talk.) We consider the simplest possible language: each party can claim either to be "keen" or to be "not keen," and for simplicity we assume that these claims are made simultaneously. We emphasize that these claims do not directly affect payoffs: they work only through affecting the other player's beliefs. In particular, they are not commitments nor are they verifiable.

To summarize, the extensive form is as follows. First, the parties simultaneously announce whether they are "keen" or "not keen"; these announcements do not directly affect either party's payoff. After observing the pair of announcements, the parties play the double auction described above. If trade takes place at price p, then a buyer with valuation $v_b$ achieves payoff $v_b-p$ and a seller with valuation $v_s$ achieves payoff $p-v_s$; if trade does not occur then payoffs are zero.

The extensive form of our formal model differs slightly from our informal story about the corporate moguls: in the formal model, the double auction is played after any pair of cheap-talk announcements, whereas in the informal atory, "Saturday evening (could) be the end of it." Intuitively, we think that a player who is made sufficiently pessimistic about the likely gains from trade will not bother to participate in bargaining. This is, of course, because there are <u>costs</u> to such participation: both disbursements (on lawyers, etc.) and opportunity costs (notably alternative negotiations foregone). We do not model these costs, since doing so would complicate the model and obscure the basic tradeoff. Even so, we could simply posit that insufficiently encouraged traders do not show up to bargain on Monday: although (absent costs) this choice is weakly dominated, it is still an equilibrium for neither side to show up (since trade cannot happen unless both sides appear). Alternatively, we can think of the parties playing the following "no-trade" equilibrium in the second stage after discouraging talk: the buyer bids $p_b = 0$, and the seller bids $p_s = 1$. Whichever choice the reader prefers, there is no trade in such a subgame.[2]

In our game, as in every cheap-talk game, there is an uncommunicative equilibrium: if cheap talk is taken to be meaningless, then parties are willing to randomize uninformatively over the possible messages. But there

are also two more interesting equilibria in which cheap talk is meaningful. In one, serious bargaining takes place only if both parties claim to be "keen"; in the other, a single such claim suffices. In both of these equilibria, serious bargaining does not occur if neither party claims to be "keen".

In the first of these equilibria with meaningful cheap talk, the Chatterjee-Samuelson equilibrium reappears: everyone claims to be "keen" except those types who are sure not to trade.[3] In this equilibrium, cheap talk is credible, but does not affect the equilibrium outcome: the mapping from type pairs to bids and probability of trade is the same as in the Chatterjee-Samuelson equilibrium, which has no cheap talk.

In the other equilibrium, however, cheap talk matters in an important way: low-value buyers and high-value sellers are willing to jeopardize continued negotiation so as to improve their bargaining position; those who have more at stake cannot afford this risk. We focus on this equilibrium because it features bargaining outcomes that could not be equilibrium behavior in the absence of talk.

We analyze our equilibrium in the standard case in which $v_s$ and $v_b$ are independently and uniformly distributed on $[0,1]$. We show in the Appendix that the following strategies for the cheap-talk stage are part of a perfect Bayesian equilibrium. Buyers above the critical type

$$y = \frac{22 + 12\sqrt{2}}{49} \approx .795$$

say "keen" while those below say "not keen". Sellers below $(1-y)$ say "keen", while those above say "not keen".

If both parties say "not keen" then the negotiation effectively ends, as discussed above. If at least one says "keen" then bargaining continues with

a (possibly asymmetric) sealed-bid double auction. If, for instance, the seller says "not keen" and the buyer says "keen" then it becomes common knowledge that the seller's type is above 1-y and the buyer's type is above y, and negotiation proceeds on that basis. Similarly, if the seller says "keen" and the buyer says "not keen" then it becomes common knowledge that the seller's type is below 1-y and the buyer's type is below y. In both of these cases, we use the Chatterjee-Samuelson equilibrium to solve the resulting bargaining game. Finally, if the buyer and the seller both say "keen" then it becomes common knowledge that the seller's type is below 1-y and the buyer's type is above y. In this case, the linear Chatterjee-Samuelson equilibrium breaks down because gains from trade are guaranteed ($y > 5/8$). Because of this, and because the subgame is symmetric, we focus on the equilibrium in which trade occurs with certainty at a price of 1/2. Formally, this is a "one-step" equilibrium (see Leininger, Linhart and Radner): the buyer bids 1/2 if his value is above 1/2 but otherwise bids 0, and the seller follows an analogous strategy. In the Appendix we state and prove Lemma 1, which describes in more detail the cheap-talk and bargaining behavior in our cheap-talk equilibrium.

Propositions 1 and 2 show that cheap-talk really matters in our equilibrium. (Proofs of these results are in the Appendix.) Proposition 1 shows that the equilibrium mapping from pairs of types $(v_b, v_s)$ to outcomes (whether trade occurs, and if so at what price) differs from any that could occur in a no-talk equilibrium, because talk achieves some correlation of bids.

Proposition 1: The equilibrium mapping from types to outcomes in our cheap-talk equilibrium cannot arise in an equilibrium without talk.

Proposition 2 shows that cheap talk matters in our equilibrium in a more fundamental way: in the {"not keen", "keen"} and {"keen", "not keen"} subgames, the bidding strategies we specify form an equilibrium only because talk changes each party's belief about the other's type.

Proposition 2: There does not exist an equilibrium without cheap talk in which sellers $v_s \in [1-y,1]$ and buyers $v_b \in [y,1]$ name the prices that they name in the {"not keen", "keen"} subgame of our cheap-talk equilibrium.

Matthews and Postlewaite also analyze cheap talk preceding a double auction. They characterize the sets of equilibrium outcomes that can be obtained through both mediated communication (as in mechanism design) and unmediated communication (as in our definition of cheap talk). Remarkably, their constructive proofs of both of these characterization results restrict attention to a simple form of unmediated-communication equilibrium: talk simply coordinates different types on different equilibria, each of which is an equilibrium without talk. This kind of coordination is impossible in an equilibrium without talk.

Proposition 1 summarizes the role of this Matthews-Postlewaite-style coordination in our cheap-talk equlibrium. Proposition 2 makes precise our claim that in our equilibrium cheap talk is credible because it allows players to trade off bargaining position against the probability of continued negotiation. Different types view this trade-off differently, and credible talk results from these differences.

## 3. Welfare

This section compares our cheap-talk equilibrium to the Chatterjee-Samuelson equilibrium. Calculation shows that our cheap-talk equilibrium yields buyer-type $v_b$ an interim payoff, evaluated before the cheap-talk phase, of:

$$W_b(v_b) = \begin{cases} 0 & \text{if } v_b \leq \frac{1}{4}y \\ \frac{1}{2}(v_b - \frac{1}{4})^2 & \text{if } \frac{1}{4}y < v_b \leq 1 - \frac{3}{4}y \\ (1-y)(v_b - \frac{1}{2} + \frac{1}{4}y) & \text{if } 1 - \frac{3}{4}y < v_b \leq y, \\ \frac{1}{2}(v_b - \frac{1}{4}y)^2 - \frac{1}{2}(\frac{7}{4}y-1)^2 & \text{if } v_b > y. \end{cases}$$

An immediate consequence is that if $y/4 = .199 < v_b \leq 1/4$, then buyer-type $v_b$ is strictly better-off than in the Chatterjee-Samuelson equilibrium. In fact, many other types are better-off in our equilibrium than in Chatterjee-Samuelson. Equating our $W_b(v_b)$ to the Chatterjee-Samuelson equivalent $W_b^{CS}(v_b) \equiv \frac{1}{2}(v_b - \frac{1}{4})^2$ yields a crossover point in the range $1 - \frac{3}{4}y < v_b < y$, given by the solution $v_b$ to

$$(1-y)(v_b - \frac{1}{2} + \frac{1}{4}y) = \frac{1}{2}(v_b - \frac{1}{4})^2,$$

which is approximately equal to .599, and indeed is between $1 - \frac{3}{4}y = .404$ and $y = .795$. Thus, all buyer-types in (.199, .599), and all seller-types in the analogous interval, are better off with cheap talk. In fact, exactly as many types strictly prefer our equilibrium as strictly prefer Chatterjee-Samuelson.

The pairs $(v_b, v_s)$ who trade in our equilibrium are illustrated in Figure 1, which also shows the corresponding region for Chatterjee-Samuelson. The

(ex-ante) probability of $(v_b, v_s)$ falling into the trading region for our equilibrium is $\frac{3}{2}y(1-y)$, or approximately .244, somewhat less than the corresponding probability (.281) for the Chatterjee-Samuelson equilibrium: our equilibrium involves less trade. Similarly, the ex-ante expected total gains from trade in our equilibrium[4] are approximately .124, less than Chatterjee-Samuelson's figure of .140.

Both of these comparisons are special cases of Myerson and Satterthwaite's general result that the Chatterjee-Samuelson equilibrium maximizes both the ex-ante probability of trade and the ex-ante gains from trade (for the independent, uniform case we have analyzed). Myerson (1985), however, convincingly argues that such ex-ante efficiency is often irrelevant, because there is seldom an opportunity to make binding arrangements ex-ante (that is, before either player knows his "type"). Myerson gives an example of an incentive-compatible mechanism (for the independent, uniform case) in which even high-value sellers (and low-value buyers) trade with positive probability, and therefore are better-off than in the Chatterjee-Samuelson equilibrium. Our cheap-talk equilibrium is in the same spirit, but is derived from an extensive-form game in which talk takes place without a mediator.[5]

## 4. Conclusion

We believe that the economic importance of costless, non-verifiable, informal communication is much greater than its role in the literature suggests. The seminal work by Crawford and Sobel is justly famous, but applications have only recently begun to appear.

This paper introduces cheap talk to bargaining games. We emphasize that cheap talk can matter in bargaining when participation is endogenous, because the parties can then use talk to trade off bargaining position against the probability of continued negotiation. This talk can matter in an important way: the cheap-talk equilibrium we analyze features bargaining outcomes that could not be equilibrium behavior in the absence of talk. Not surprisingly, this intuition does not depend on anything as specific as a sealed-bid double auction. See Farrell and Gibbons (1986) for a more general discussion of these issues.

Cheap talk can be important in many other economic settings as well. Farrell (1987), for instance, studies cheap talk between potential entrants in a natural monopoly, Farrell and Saloner (1985) consider cheap talk between potential adopters of a new technology, Forges (1986) analyzes cheap talk in a hiring and job-assignment game, Gibbons (1987) models interest arbitration as a cheap-talk game, Matthews (1987) describes presidential rhetoric as a cheap-talk veto threat in the budgetary process, and Sobel (1985) develops a theory of credibility in finitely repeated relationships. The fundamental insight that cheap talk can be credible in variable-sum games, combined with the ubiquity of such talk, suggests that a rich collection of other applications lies ahead.

## Appendix

In this Appendix we adapt the Chatterjee-Samuelson analysis to suit our purposes, and then use the results to derive the equilibrium value of y. We give a formal statement (and proof) of the equilibrium cheap-talk and bargaining behavior in Lemma 1. We also prove Propositions 1 and 2.

Consider a sealed-bid double auction in which it is common knowledge that the seller-type $v_s$ is uniformly distributed on $[\underline{v}_s, \bar{v}_s]$ and buyer-type $v_b$ is independently and uniformly distributed on $[\underline{v}_b, \bar{v}_b]$. Both parties name prices, $p_s$ and $p_b$, and trade occurs at the average of the two prices if the buyer's price exceeds the seller's.

As Chatterjee and Samuelson show, an essential part of a bounded, monotone, and differentiable equilibrium is the solution of a linked pair of differential equations, and one solution (on which we and they focus) is linear:

$$\tilde{p}_s(v_s) = \frac{2}{3} v_s + \frac{1}{4} \bar{v}_b + \frac{1}{12} \underline{v}_s, \text{ and}$$

(1)

$$\tilde{p}_b(v_b) = \frac{2}{3} v_b + \frac{1}{4} \underline{v}_s + \frac{1}{12} \bar{v}_b.$$

Three cases must be considered. First, if the use of these strategies would imply that no type of either party is sure to trade (that is, $\tilde{p}_b(\bar{v}_b) \leq \tilde{p}_s(\bar{v}_s)$ and $\tilde{p}_s(\underline{v}_s) \geq \tilde{p}_b(\underline{v}_b)$), then the equilibrium strategies are $p_s(v_s) = \tilde{p}_s(v_s)$ and $p_b(v_b) = \tilde{p}_b(v_b)$. Second, if the use of these strategies would make some types of at least one party sure to trade, but would not make all types of both parties sure to trade, then the buyer-type $v_b$ names the price $p_b(v_b) = \min(\tilde{p}_b(v_b), \tilde{p}_s(\bar{v}_s))$ and the seller-type $v_s$ names the price $p_s(v_s) = \max(\tilde{p}_s(v_s), \tilde{p}_b(\underline{v}_b))$. Third, if the use of these strategies would make all types of both players sure to trade, then the Chatterjee-Samuelson

equilibrium breaks down. A continuum of equilibria still exists, but these equilibria are unrelated to the bidding strategies described in (1). In each of these equilibria, all types of both parties name a given price in the interval $[\bar{v}_s, \underline{v}_b]$ and trade occurs with certainty. We deal with this case below.

When no seller-type is sure to trade, calculation shows that the buyer's interim payoff is:

$$(2) \qquad U_b(v_b;\ [\underline{v}_s,\bar{v}_s],[\underline{v}_b,\bar{v}_b]) = \begin{cases} 0 & \text{if } v_b \le \underline{\beta}, \\ \dfrac{(v_b-\underline{\beta})^2}{2(\bar{v}_s-\underline{v}_s)} & \text{if } \underline{\beta} < v_b \le \bar{\beta}, \\ v_b - \bar{\beta} + \dfrac{\bar{v}_s - \underline{v}_s}{2} & \text{if } v_b > \bar{\beta}, \end{cases}$$

where $\Delta = (\bar{v}_b - \underline{v}_s)/4$, $\underline{\beta} = \underline{v}_s + \Delta$, and $\bar{\beta} = \bar{v}_s + \Delta$. (In this notation, no seller-type is sure to trade when $\underline{v}_b < \underline{\beta}$.) The three cases in (2) correspond to the cases in which the buyer, given $v_b$ and the supports of the players' types, is sure not to trade, might trade, or is sure to trade, respectively.

When some but not all seller-types are sure to trade (i.e., $\underline{\beta} < \underline{v}_b < \bar{\beta}$), an interval of seller-types trade with the lowest buyer-type. The interim payoff to $\underline{v}_b$ is then

$$(3) \qquad \underline{U}_b \equiv U_b(\underline{v}_b;[\underline{v}_s,\bar{v}_s],[\underline{v}_b,\bar{v}_b]) = \frac{(\underline{v}_b-\underline{v}_s-\Delta)^2}{3(\bar{v}_s-\underline{v}_s)},$$

and the interim payoff for other buyer types is

$$(4) \qquad U_b(v_b;\ [\underline{v}_s,\bar{v}_s],[\underline{v}_b,\bar{v}_b]) = \begin{cases} \dfrac{(v_b-\underline{v}_s-\Delta)^2}{2(\bar{v}_s-\underline{v}_s)} - \dfrac{1}{2}\underline{U}_b & \text{if } v_b \le \bar{\beta}, \\ v_b - \bar{\beta} - \dfrac{1}{2}\underline{U}_b + \dfrac{\bar{v}_s - \underline{v}_s}{2} & \text{if } v_b > \bar{\beta}. \end{cases}$$

Finally, when all seller-types are sure to trade ($\bar{\beta} < \underline{v}_b$), then all buyer-types also are sure to trade, and the Chatterjee-Samuelson equilibrium breaks down: the bids given by (1) are irrelevant, and the strategies $p_b(v_b) = \tilde{p}_s(\bar{v}_s)$ and $p_s(v_s) = \tilde{p}_b(\underline{v}_b)$ are <u>not</u> an equilibrium. Without proposing a general theory for this problem, we note that the subgame $\{v_s \in [0,1-y],\ v_b \in [y,1]\}$ is symmetric about $\frac{1}{2}$ and so (when $y > \frac{1}{2}$) it is natural to assume that trade will occur with certainty at a price of $\frac{1}{2}$. Then a buyer-type $v_b \geq \frac{1}{2}$ gets a payoff $(v_b - \frac{1}{2})$. As noted in the text, a "one-step" equilibrium at price $\frac{1}{2}$ formalizes this outcome.

As described in the text, an equilibrium value of y must satisfy the necessary condition

$$(5) \qquad (1-y)U_b(y;[0,1-y],[y,1]) + yU_b(y;[1-y,1],[y,1])$$

$$= (1-y)U_b(y;[0,1-y],[0,y]),$$

since the left-hand side represents buyer-type y's expected payoff if he says "keen" and the right-hand side represents his expected payoff if he says "not keen."

The first term on the left-hand side of (5) is strictly less than the right-hand side, because the only difference is that in the first term the seller is more optimistic about the buyer's type. Therefore the second term on the left-hand side is strictly positive, so the buyer-type y trades at least sometimes in that subgame ($y > \underline{\beta} = 1 - \frac{3}{4}y$, or $y > \frac{4}{7}$), and some seller-types trade for sure. On the other hand, since in this subgame $\bar{\beta} > 1$, not all seller-types trade for sure. Thus (3) applies in the second term of (5).

On the right-hand side of (5), which involves the subgame $\{v_s \in [0,1-y],\ v_b \in [0,y]\}$, the critical type $\bar{\beta}$ is equal to $1-\frac{3}{4}y$, so y trades for sure in that subgame, since $y > \frac{4}{7}$ implies $y > \bar{\beta}$. But $\underline{v}_b = 0 < \underline{\beta} = \frac{1}{4}y$, so no seller-type is sure to trade and the bottom case of (2) applies.

Finally, in the first term on the left-hand side of (5), involving the subgame $\{v_s \in [0,1-y],\ v_b \in [y,1]\}$ (in which both players are keen), $\bar{\beta} = \frac{5}{4} - y$. So if $y > \frac{5}{4} - y$, or $y > \frac{5}{8}$, then y trades for sure when both agents are keen. This means that all types of both agents trade for sure, and the Chatterjee-Samuelson analysis breaks down; as discussed in the text, we consider the one-step equilibrium in which trade occurs with certainty at price $\frac{1}{2}$.

Substituting all this into (5) yields

$$(1-y)(y - \tfrac{1}{2}) + \tfrac{1}{3}y(\tfrac{7}{4}y-1)^2 = (1-y)(\tfrac{5}{4}y - \tfrac{1}{2}),$$

which has solutions

$$y = \{22 \pm 12\sqrt{2}\}/49$$
$$\approx .103 \text{ or } .795$$

Since the analysis of the second term of (5) proved that $y > \frac{4}{7}$, the solution is $y \approx .795$, which exceeds $\frac{5}{8}$, confirming that the Chatterjee-Samuelson equilibrium indeed breaks down when both parties say "keen."

This completes our derivation of the equilibrium value of y. We next state and prove Lemma 1, which describes our equilibrium. Finally, we prove Propositions 1 and 2.

Lemma 1: The following behavior defines the equilibrium path of a perfect Bayesian equilibrium in our two-stage bargaining game. (Our proof does not require explicit descriptions of the optimal bids following deviations in the

cheap-talk phase, so we omit these bids from the description of equilibrium.)

Buyer: If $v_b < y$ then

(i) say "not keen" in the first stage, and

(ii) if the seller says "not keen" in the first stage then bid $p_b = 0$ in the second stage, but if the seller says "keen" in the first stage then bid

$$p_b = \min\{\tfrac{2}{3}v_b + \tfrac{1}{12}y,\ \tfrac{2}{3} - \tfrac{5}{12}y\}$$

in the second stage.

If $v_b \geq y$ then

(i) say "keen" in the first stage, and

(ii) if the seller says "keen" in the first stage then bid $p_b = 1/2$ in the second stage, but if the seller says "not keen" in the first stage then bid

$$p_b = \tfrac{2}{3}v_b + \tfrac{1}{4}(1-y) + \tfrac{1}{12}$$

in the second stage.

Seller: If $v_s > 1-y$ then

(i) say "not keen" in the first stage, and

(ii) if the buyer says "not keen" in the first stage then bid $p_s = 1$ in the second stage, but if the buyer says "keen" in the first stage then bid

$$p_s = \max\{\tfrac{2}{3}v_s + \tfrac{1}{4} + \tfrac{1}{12}(1-y),\ \tfrac{1}{3} + \tfrac{5}{12}y\}$$

in the second stage.

If $v_s \leq 1-y$ then

(i) say "keen" in the first stage, and

(ii) if the buyer says "keen" in the first stage then bid $p_s = \frac{1}{2}$ in the second stage, but if the buyer says "not keen" in the first stage then bid

$$p_s = \frac{2}{3}v_s + \frac{1}{4}y$$

in the second stage.

Proof. The equilibrium bidding strategies in the {"keen", "not keen"} and {"not keen", "keen"} subgames follow from the analysis in the first part of this Appendix. This means that the bidding strategies specified in the Lemma are sequentially rational. It therefore suffices to show that the necessary condition (5) is also sufficient. This is a direct result of the fact that higher buyer-types are more concerned with the probability of trade than are lower buyer-types.

Formally, define the difference between the payoffs from saying "keen" and saying "not keen," given subsequent optimal behavior by a buyer of type $v_b$, as

$$\begin{aligned} D(v_b) \equiv\ & (1-y)W_b(v_b;[0,1-y],[y,1]) \\ & + yW_b(v_b;[1-y,1],[y,1]) \\ & - (1-y)W_b(v_b;[0,1-y],[0,y]), \end{aligned}$$

where $W_b(v_b;[\underline{v}_s,\bar{v}_s],[\underline{v}_b,\bar{v}_b])$ is the expected payoff to buyer-type $v_b$ from making the optimal bid when the seller's type is uniformly distributed on

$[\underline{v}_s, \bar{v}_s]$ and the seller <u>believes</u> that the buyer's type is independently and uniformly distributed on $[\underline{v}_b, \bar{v}_b]$. Note that $W_b$ is defined even if $v_b$ falls outside the interval $[\underline{v}_b, \bar{v}_b]$ that the seller believes contains the buyer's type, as will occur off the equilibrium path. If $v_b \in [\underline{v}_b, \bar{v}_b]$, however, then

$$W_b(v_b; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]) = U_b(v_b; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]),$$

as will occur in equilibrium.

We must show that $D(v_b) \geq 0$ for $v_b \geq y$ and that $D(v_b) \leq 0$ for $v_b \leq y$. This will show that our proposed equilibrium is indeed an equilibrium -- that is, that no buyer can gain by deviating in the cheap-talk stage and then submitting an unexpected bid. (A similar proof works for the seller.) We do this by showing that

(i) for $v_b \leq 1/2$, $D(v_b) \leq 0$, and

(ii) for $v_b > 1/2$, $D'(v_b) \geq 0$.

Since $y > \frac{1}{2}$ and since $D(y) = 0$, (i) and (ii) prove our proposition. Note that the arguments below do not require explicit descriptions of optimal bids following deviations in the cheap-talk phase.

Consider first $v_b \leq 1/2$. Since the seller bids 1/2 after {"keen", "keen"}, the first term of $D(v_b)$ vanishes. Moreover, the seller's minimum bid after {"not keen", "keen"} is

$$p_s(1-y) = \max \{\tilde{p}_s(1-y), \tilde{p}_b(y)\}$$

$$= \tilde{p}_b(y) = \frac{1}{3} + \frac{5}{12}y,$$

and since $\frac{1}{3} + \frac{5}{12}y > 1/2$, the second term of $D(v_b)$ also vanishes. Hence, it is immediate that $D(v_b) \leq 0$ for $v_b \leq 1/2$.

Now consider the derivative $D'(v_b)$, for $v_b \geq 1/2$. Note that the derivative of the interim payoff is equal to the probability of trade (by the envelope theorem), even off the equilibrium path. Since buyer-types $v_b \geq 1/2$ trade for sure in the {"keen", "keen"} subgame (since the seller bids 1/2 for sure), the derivative of the first term of $D(v_b)$ is $(1-y)$. The derivative of the third (negative) term is $[-(1-y)]$ times a probability. So, for $v_b \geq 1/2$,

$$D'(v_b) \geq \frac{d}{dv_b}\{yW_b(v_b;[1-y,1],[y,1]\} \geq 0.$$

Q.E.D.

Proposition 1: The equilibrium mapping from types to outcomes in our cheap-talk equilibrium cannot arise in an equilibrium without talk.

Proof: Consider a buyer of type $v_b > y$. In equilibrium he sometimes bids 1/2 (following "keen" from the seller) and sometimes bids

$$\tilde{p}_b(v_b;[1-y,1],[y,1]) = \frac{2}{3}v_b + \frac{1}{4}(1-y) + \frac{1}{12}$$

$$= \frac{1}{3} + \frac{2}{3}v_b - \frac{1}{4}y$$

(following "not keen" from the seller.) Note that the latter bid exceeds 1/2. Dividing his bids into the two classes, "bids = 1/2" and "bids > 1/2", we see that the frequency of the first class is $(1-y)$ (i.e., the probability that the seller says "keen.") We notice also that the seller facing this buyer sometimes bids 1/2 and sometimes bids

$$p_s(v_s) = \max\{\tilde{p}_s(v_s;[1-y,1],[y,1]),\ \tilde{p}_b(y;[1-y,1],[y,1])\}$$

$$\geq \tilde{p}_b(y;[1-,1],[y,1])$$

$$= \frac{1}{3} + (\frac{2}{3} - \frac{1}{4})y = \frac{1}{3} + \frac{5}{12}y > \frac{1}{2}\ .$$

Given knowledge only of the buyer's type $v_b > y$, therefore, we see a joint distribution of buyer's and seller's bids as follows:

| | | $p_s$ | |
|---|---|---|---|
| | | 1/2 | >1/2 |
| $p_b$ | 1/2 | (1-y) | 0 |
| | >1/2 | 0 | y |

But such correlation in bids would be unattainable in a Nash equilibrium without cheap talk. This shows that the equilibrium <u>strategies</u> (maps from types to bids) could not arise without talk.

We now extend this argument to show that the equilibrium mapping from types to outcomes (whether trade occurs and if so at what price) is unattainable in a Nash equilibrium without cheap talk. Consider the buyer types $v_b \in [y,1]$, who say "keen" in the talk phase. If the seller says "not keen" then trade occurs with positive probability, and if trade occurs then (by Lemma 1) the price moves in $v_b$. If the seller says "keen", however, then trade occurs with probability one at price 1/2, which is independent of $v_b$. Thus, the price at which trade occurs either does or does not depend on $v_b$ according to the value of $v_s$. Clearly, the equilibrium map from types to outcomes in a double auction without talk cannot behave in this way.

Q.E.D.

Lemma 2: In any equilibrium without talk, the seller's bid $p_s$ is a weakly increasing function of his type $v_s$.

Proof: Given any seller's bid $p_s$, the distribution $F_b(\cdot)$ of the buyer's bid $p_b$ determines a probability of trade, $\pi = 1-F_b(p_s)$, and an expected price conditional on trade,

$$e = E_{p_b}\{(p_b+p_s)/2 \mid p_b \geq p_s\}.$$

Thus in choosing a bid $p_s$ the seller is choosing a point in $(e,\pi)$ space. The relationship between $p_s$ and e is strictly monotone, moreover.

In terms of $(e,\pi)$, the seller's expected payoff is just $\pi(e-v_s)$. Therefore sellers with higher $v_s$ have steeper indifference curves in $(e,\pi)$ space. This means that if $v_s$ prefers $(e,\pi)$ to $(e',\pi')$, and $e' < e$ then so does $v_s' > v_s$.

Let $e^*(v_s)$ be the optimal choice of e on the available locus for seller-type $v_s$. Then our observation concerning the slopes of the indifference curves for different types implies that $e^*$ is (weakly) increasing in $v_s$. Consequently, so is the bid $p_s$.

Q.E.D.

Proposition 2. There does not exist an equilibrium without cheap talk in which sellers $v_s \in [1-y,1]$ and buyers $v_b \in [y,1]$ name the prices that they name in the {"not keen", "keen"} subgame of our cheap-talk equilibrium.

Proof. Suppose the buyer believes that the seller's bid is distributed as $F(p_s)$. Define $Z(v_b,p_b,F)$ to be the expected payoff to buyer-type $v_b$ from bidding $p_b$ when $p_s$ is distributed as F. This buyer-type's optimal bid,

$p_b(v_b;F)$, maximizes $Z(v_b,p_b,F)$.

Let F be the buyer's belief about $p_s$ in the {"not keen", "keen"} subgame of our cheap-talk equilibrium. Then from Lemma 1 we have that

$$p_b(v_b;F) = \frac{2}{3}v_b + \frac{1}{4}(1-y) + \frac{1}{12}$$

and that for $v_b \in (y,1)$ the derivative

$$\left.\frac{\partial Z(v_b,p_b,F)}{\partial p_b}\right|_{p_b(v_b;F)}$$

exists and equals zero.

Now let G be any distribution with support bounded above by $p_s(1-y)$, the lowest seller bid in our {"not keen", "keen"} subgame. A simple computation shows that $p_b(v_b;F)$ can be rewritten as

$$p_b(v_b;F) = p_s(1-y) + (2/3)(v_b-y),$$

which exceeds $p_s(1-y)$ for $v_b > y$. Since $G[p_s(1-y)] = 1$ and $p_s(1-y) < p_b(v_b;F)$,

$$\left.\frac{\partial Z(v_b,p_b,G)}{\partial p_b}\right|_{p_b(v_b;F)}$$

exists and is strictly negative, for $v_b > y$.

Finally, suppose that there were an equilibrium without talk in which the relevant buyers and sellers name the required prices. Then by Lemma 2, the prices named by sellers $v_s \in [0,1-y]$ cannot exceed $p_s(1-y)$. The buyer's belief about $p_s$ in such an equilibrium can be described by the distribution

$$H = \lambda F + (1-\lambda)G$$

for some $\lambda \in (0,1)$ and distributions F and G as described above. Note that

$$Z(v_b, p_b, H) = \lambda Z(v_b, p_b, F) + (1-\lambda) Z(v_b, p_b, G).$$

Hence,

$$\left.\frac{\partial Z(v_b, p_b, H)}{\partial p_b}\right|_{p_b(v_b;F)} < 0,$$

so $p_b(v_b;F)$ is no longer an equilibrium bid for buyer-type $v_b > y$.

Q.E.D.

## Footnotes

[1]For those who miss the lawyers, consider the commitment necessary to play even this simple game: what, for instance, stops one party from reneging on his offer in order to capitalize on the information conveyed by the other party's offer?

[2]A third rationale for there being no trade after both players say of "not keen" is that they need to coordinate on when and where to meet on Monday, and an attempt to arrange a meeting belies a party's claim that he is "not keen". Note that this is not the same as saying that talk determines whether the parties can meet; such talk would not be cheap talk. Here the set of times and places available for a meeting is independent of the talk, but is large enough and sufficiently lacking in focal points that meeting without agreement is unlikely.

[3]In the standard case in which $v_b$ and $v_s$ are independently and uniformly distributed on $[0,1]$, all buyers with $v_b > 1/4$ and all sellers with $v_s < 3/4$ claim to be "keen". Strictly, the other types of buyers and sellers, who will not trade, may say anything. But if there are any costs of serious bargaining, then they must say "not keen".

[4]The ex-ante expected total gains from trade in our equilibrium are given by

$$(1-y)^2 \cdot \frac{8-11y}{12} + (1-\tfrac{1}{4}y)^2 \cdot \frac{2+y}{12} - \frac{1}{3}(\tfrac{3}{4}y)^3 + \frac{1}{2}(1-y)[y^2 - (1-\tfrac{3}{4}y)^2].$$

[5]Of course, the subsequent sealed-bid double auction does require a mediator, both to accept simultaneous reports and to force the parties to walk away if trade is not prescribed (even if trade would be efficient). But such a game is hardly central to the trade-off between bargaining position and the probability of continued negotiation that allows cheap talk to be credible in our analysis.

## References

Chatterjee, K. and W. Samuelson (1983); "Bargaining under Incomplete Information," Operations Research; 31, 835-851.

Crawford, V. and J. Sobel (1982); "Strategic Information Transmission," Econometrica; 50, 1431-1451.

Farrell, J. (1987); "Cheap Talk, Coordination, and Entry," Rand Journal of Economics, 18; 34-39.

——— and R. Gibbons (1986); "Cheap Talk in Bargaining Games," M.I.T. Working Paper #422, June.

——— and G. Saloner (1985); "Standardization, Compatibility, and Innovation," Rand Journal of Economics; 16, 70-83.

Forges, F. (1986); "Negotiation without a Deadline: a Job Market Example," CORE D.P. #8639.

Fudenberg, D. and J. Tirole (1983); "Sequential Bargaining with Incomplete Information," Review of Economic Studies; 50, 221-247.

Gibbons, R. (1987); "Learning in Equilibrium Models of Arbitration," M.I.T. mimeo, September.

Leininger, W., P. Linhart, and R. Radner (1986); "The Sealed Bid Mechanism for Bargaining with Incomplete Information," AT&T Bell Laboratories, mimeo, March.

Matthews, S. (1987); "Veto Threats: Rhetoric in a Bargaining Game," University of Pennsylvania mimeo, February.

——— and A. Postlewaite (1987); "Pre-Play Communication in Two-Person Sealed-Bid Double Auctions," Pennsylvania CARESS Working Paper #87-12R.

Myerson, R. (1985); "Analysis of Two Bargaining Problems with Incomplete Information," in A. Roth (ed.), Game Theoretic Models of Bargaining; Cambridge University Press.

——— and M. Satterthwaite (1983); "Efficient Mechanisms for Bilateral Trading," JET; 29, 265-281.

Satterthwaite, M. and S. Williams (1987); "Bilateral Trade with the Sealed Bid Double Auction: Existence and Efficiency," Northwestern, CMSEMS DP #723.

Sobel, J. (1985); "A Theory of Credibility," Review of Economic Studies; 52, 557-573.

——— and I. Takahashi (1983); "A Multistage Model of Bargaining," Review of Economic Studies; 50, 411-426.

Spence, A. M. (1974); Market Signaling; Harvard University Press, Cambridge.

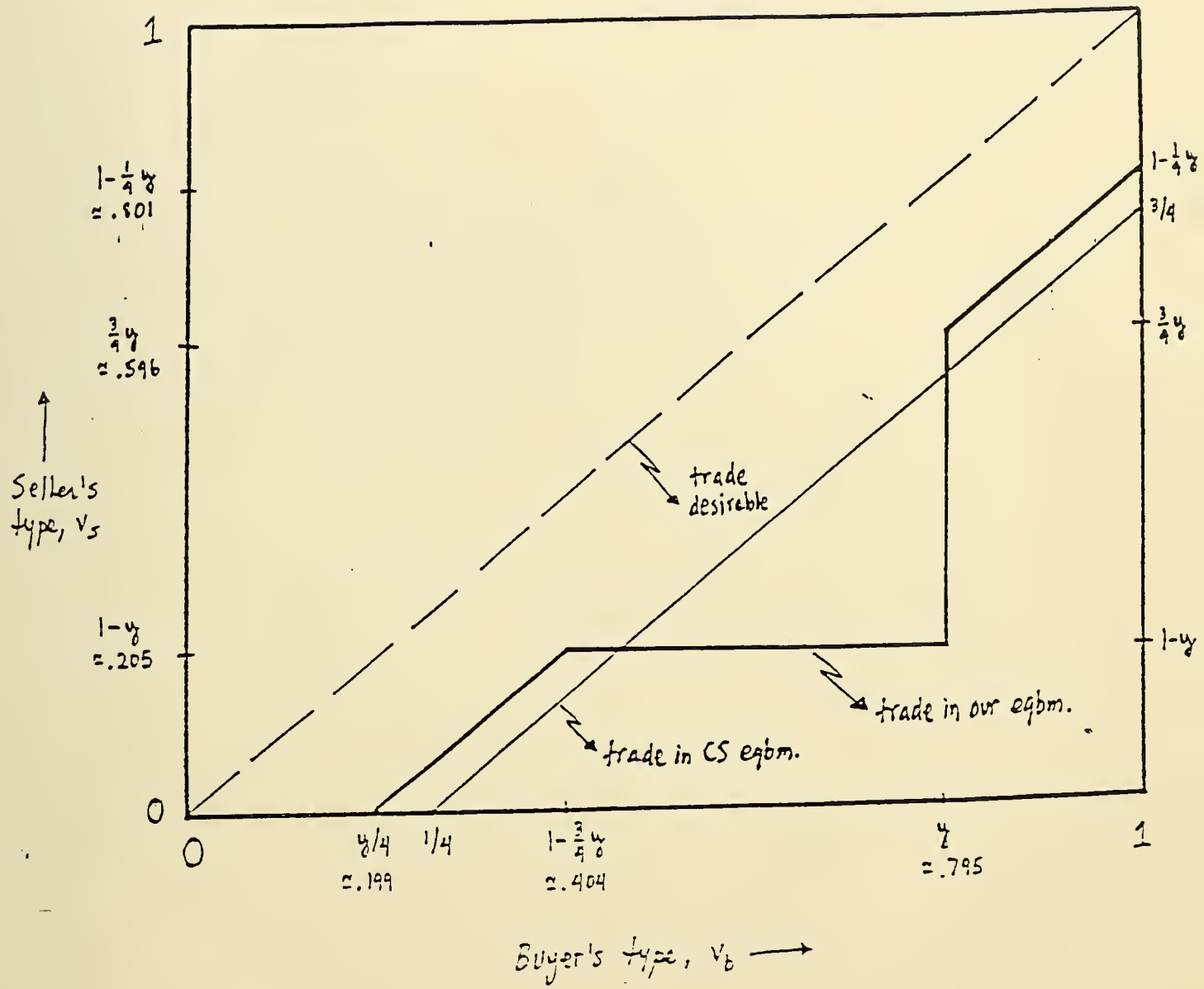
1
$1-\frac{1}{4}\gamma$
≈ .801
$\frac{3}{4}\gamma$
≈ .596
$1-\gamma$
≈ .205
0
Seller's type, $v_s$
trade desireble
trade in our eqbm.
trade in CS eqbm.
0
$\gamma/4$
≈ .199
1/4
$1-\frac{3}{4}\gamma$
≈ .404
$\gamma$
≈ .795
1
Buyer's type, $v_b$ →
$1-\frac{1}{4}\gamma$
3/4
$\frac{3}{4}\gamma$
$1-\gamma$

FIGURE 1

5582 092

Date Due

OC 23 '88

MR 25 '89

APR 21 1991

Lib-26-67

MIT LIBRARIES
3 9080 005 336 794